SOME NOTES ON THE SAMPLE COMPLEXITY OF APPROXIMATE CHANNEL SIMULATION

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IN COLLABORATION WITH

STUDYING CHANNEL SIMULATION HELPS APPROXIMATE SAMPLING!

APPROXIMATE SAMPLING

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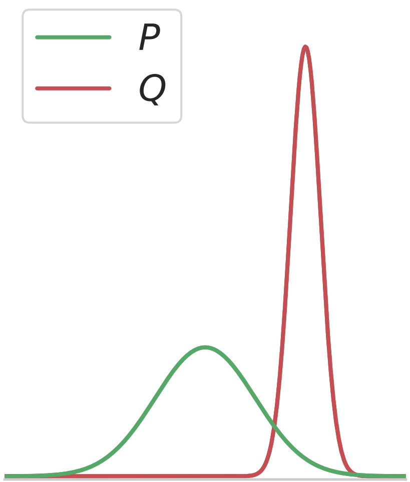

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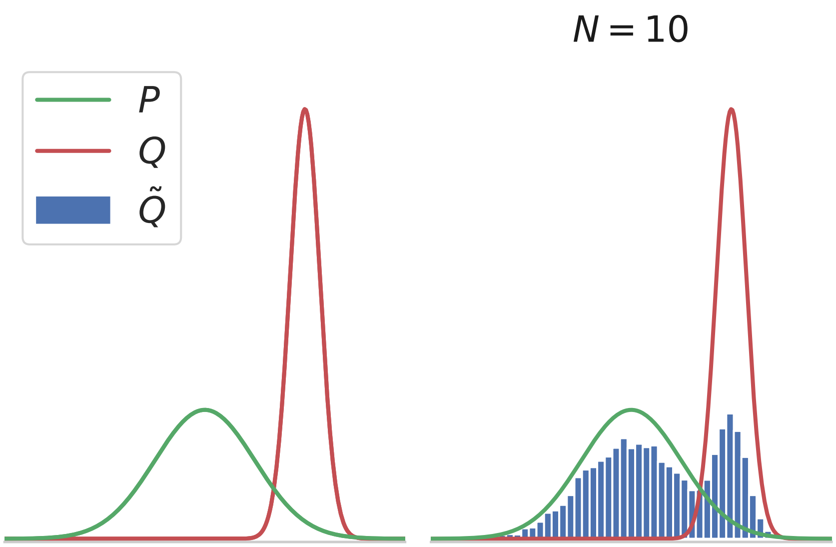

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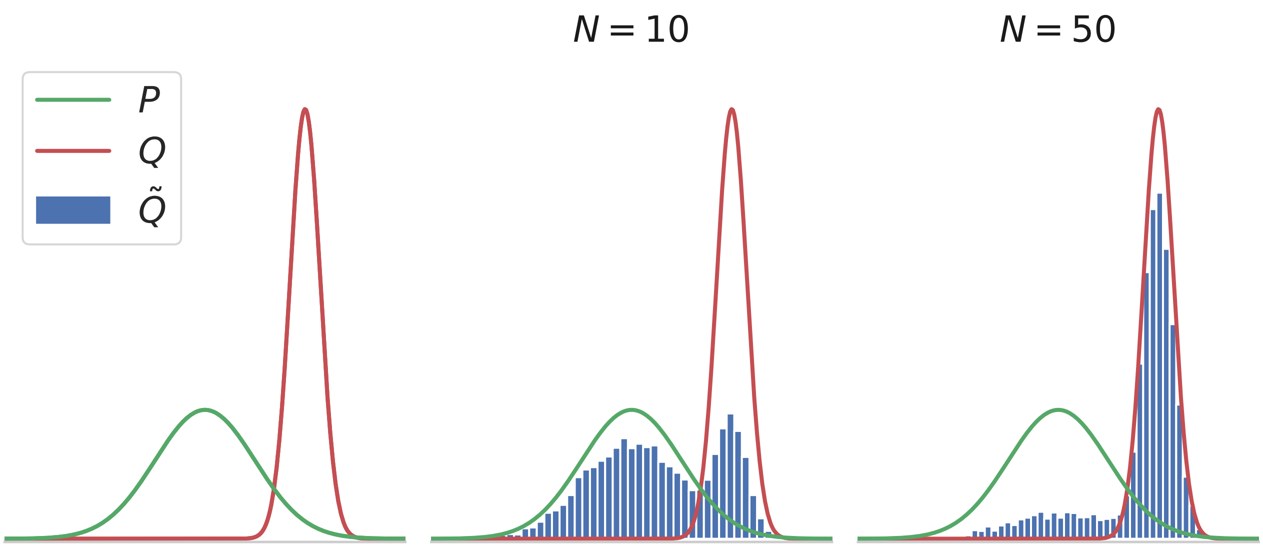

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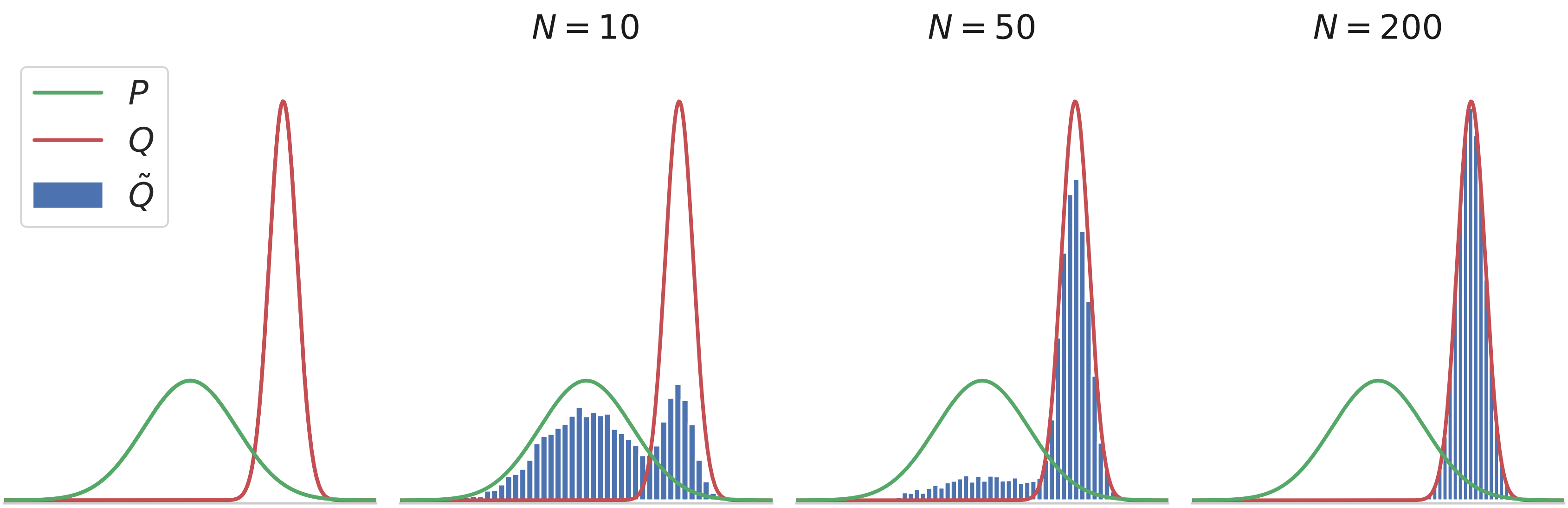

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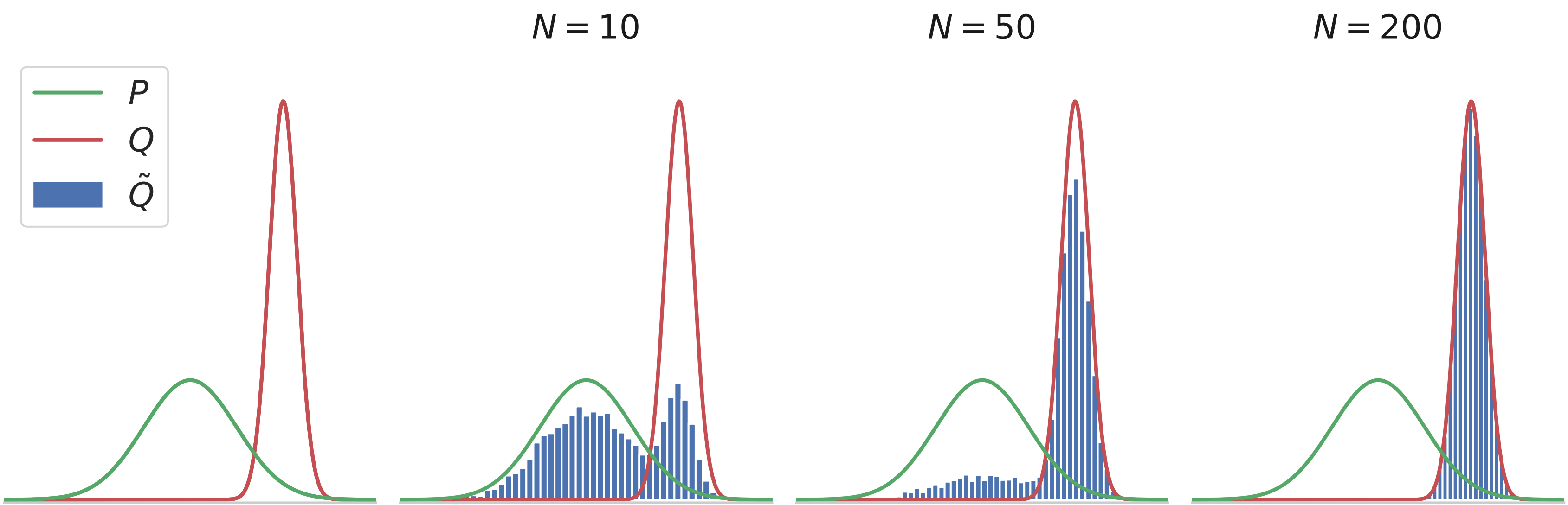

If we want $TV[Q\,||\tilde{Q}] \leq \epsilon$, how big should N be?

IDEA: USE EXACT SAMPLER

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K - selection rule of exact sampler

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Block and Polyanskiy (2023):

$$N \geq \frac{2}{1-\epsilon} \log\left(\frac{2}{\epsilon}\right) \exp\left(\frac{4 \cdot KL[Q\,||\,P]}{\epsilon}\right)$$

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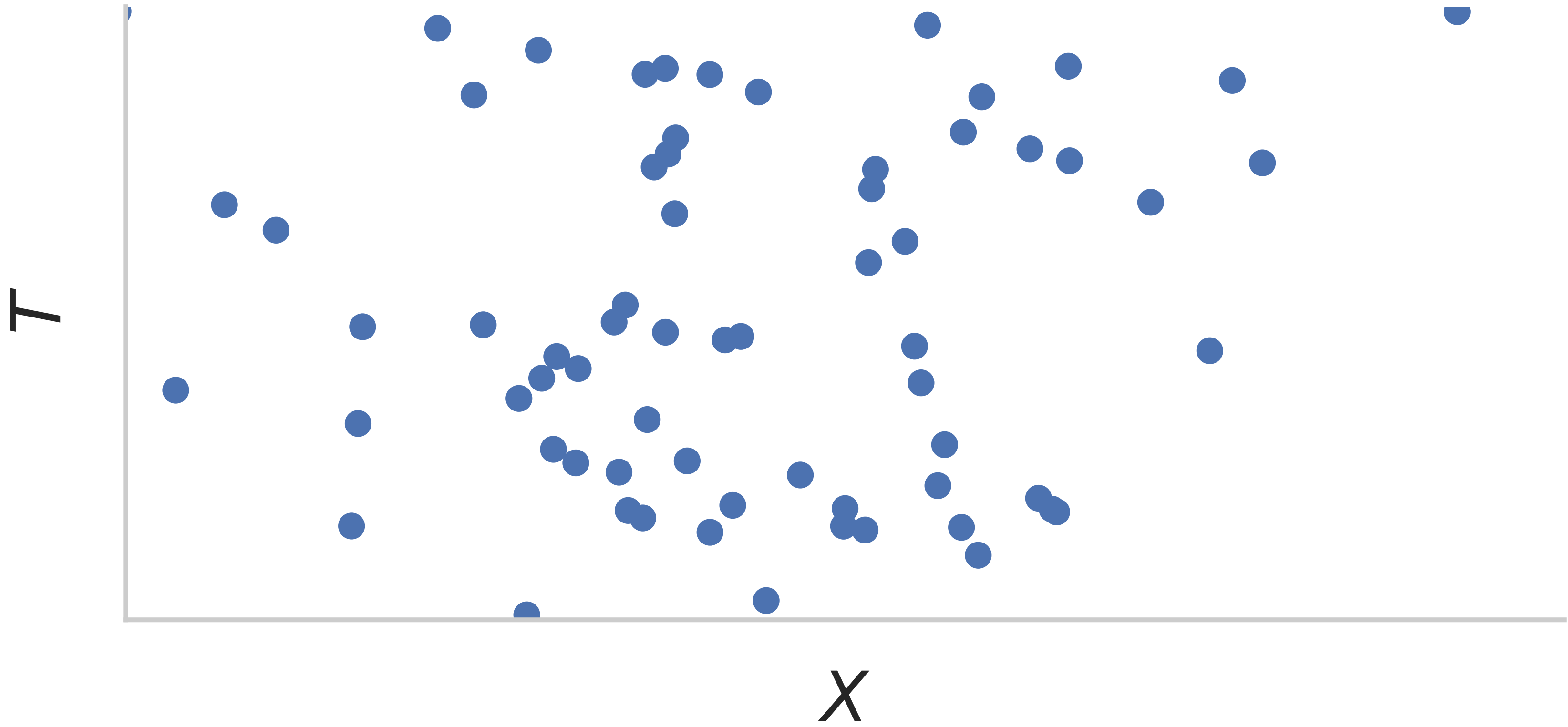

7.1

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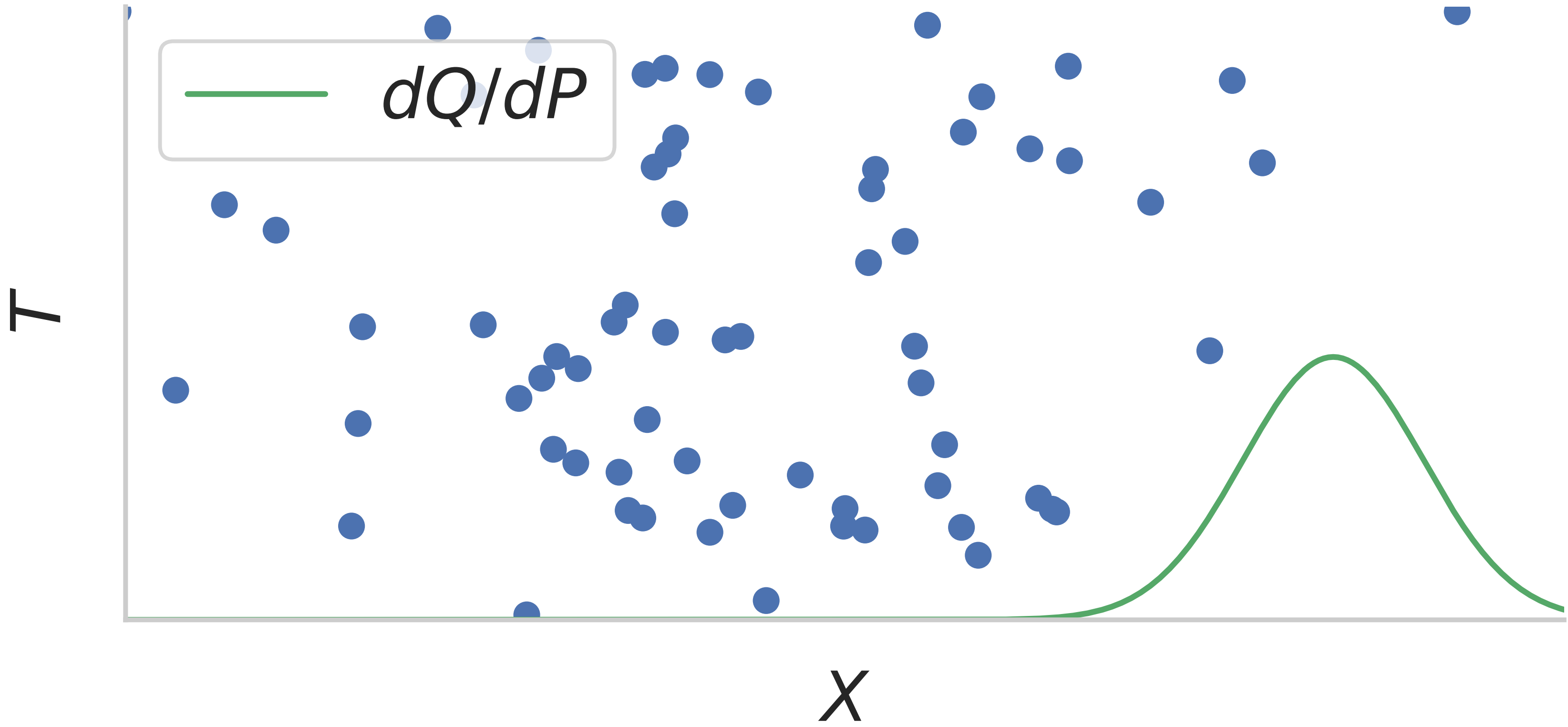
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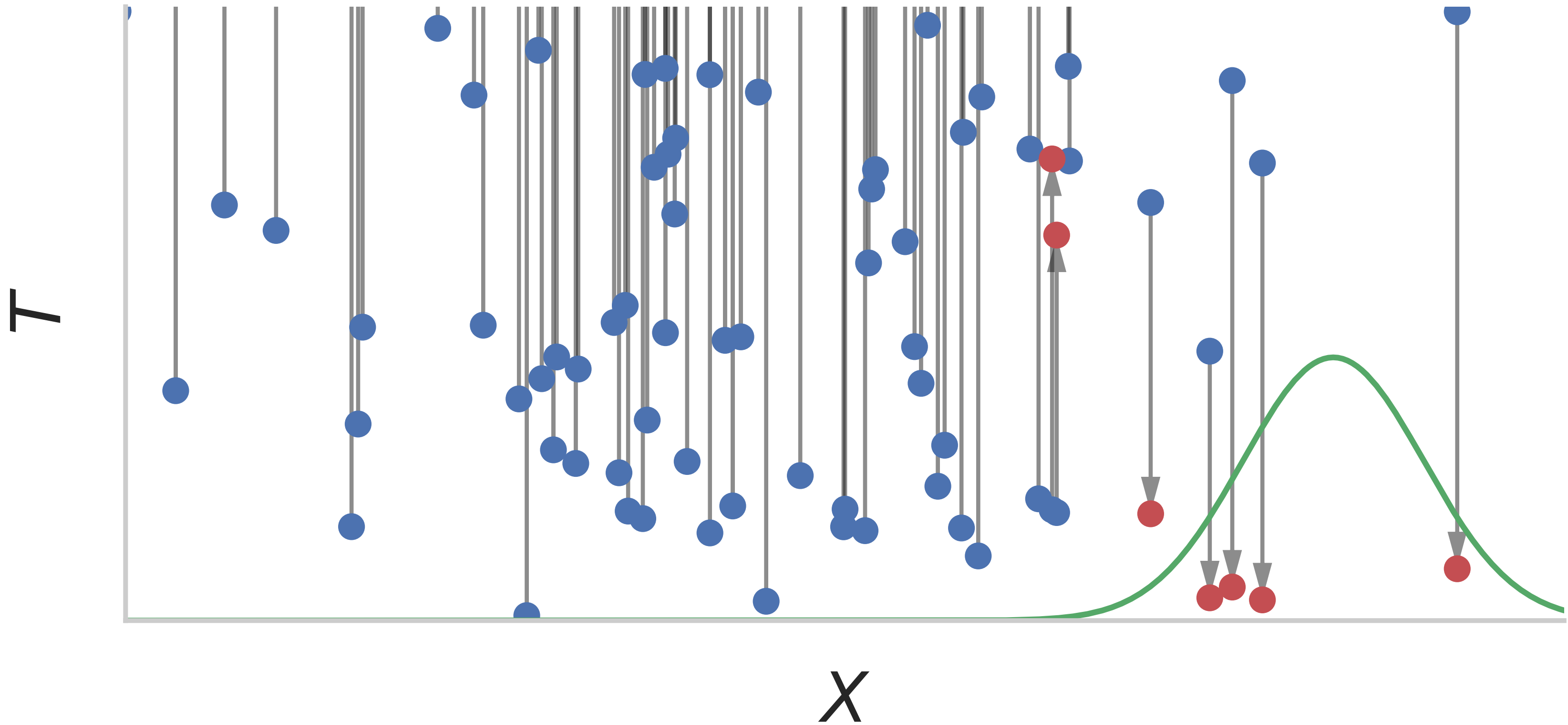
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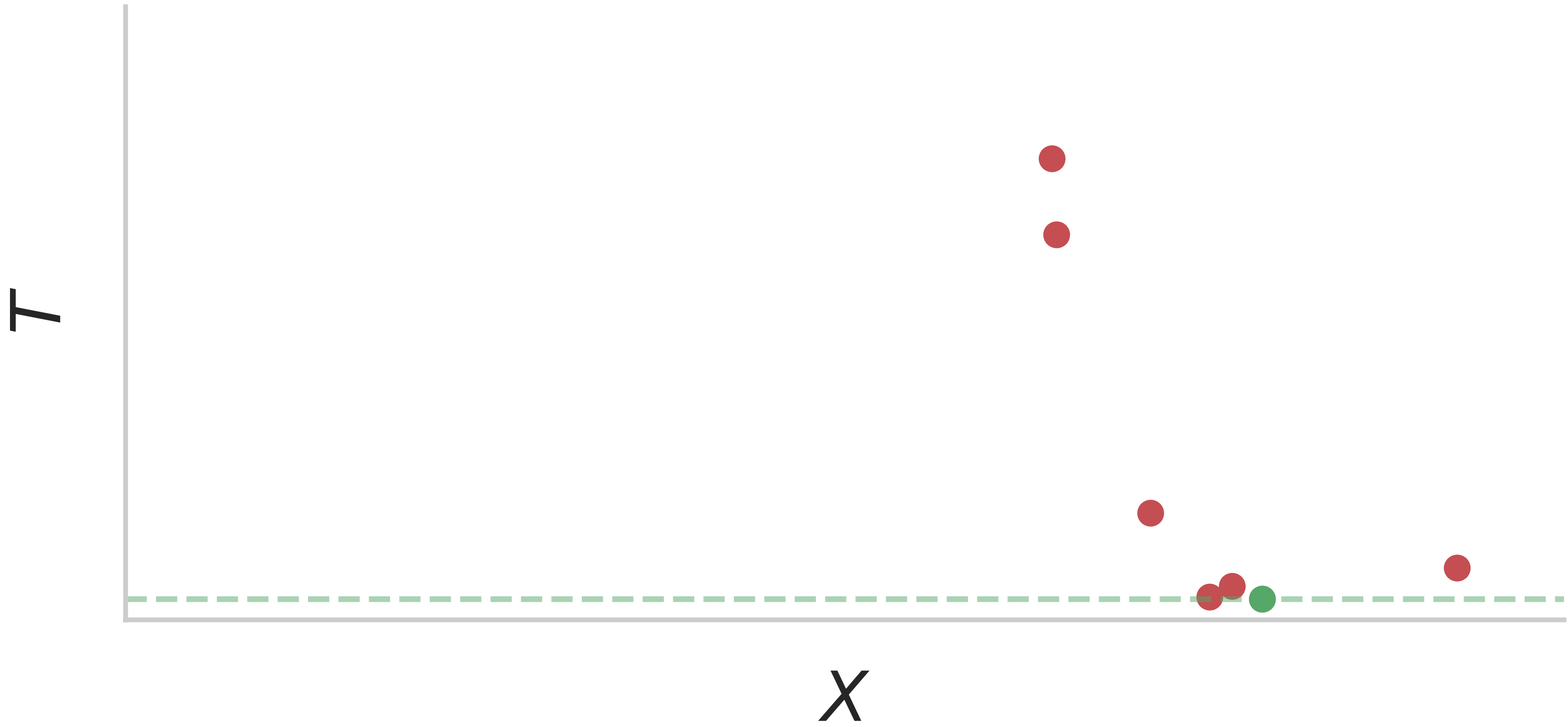

7.4

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Li and El Gamal (2018):

$$\mathbb{E}[\log K] \leq KL[Q\,||\,P] + \mathcal{O}(1)$$

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Furthermore:

$$\mathbb{H}[K] \leq KL[Q \,||\, P] + \log(KL[Q \,||\, P] + 1) + \mathcal{O}(1)$$

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3. See paper for additional sampling complexity boun

REFERENCES

- Block, A., & Polyanskiy, Y. (2023, July). The sample complexity of approximate rejection sampling with applications to smoothed online learning. In The Thirty Sixth Annual Conference on Learning Theory (pp. 228-273).
- Li, C. T., & El Gamal, A. (2018). Strong functional representation lemma and applications to coding theorems. IEEE Transactions on Information Theory, 64(11), 6967-6978.

MORE GENERAL BOUNDS

Block and Polyanskiy (2023):

$$N \geq \frac{2}{1-\epsilon}\log\left(\frac{2}{\epsilon}\right)(f')^{-1}\left(\frac{4 \cdot D_f[Q \,||\, P]}{\epsilon}\right)$$

Ours: for $\gamma \in (0,1)$

$$N \geq \log\left(\frac{1}{(1-\gamma)\epsilon}\right)(f')^{-1}\left(\frac{D_f[Q \,||\, P]}{\gamma\epsilon}\right)$$